8.2 Substituting Numbers for Variables (DOK 2)

Most algebra problems involve using variables for numbers. In this section, we will substitute numbers for variables to solve.

Example 1: In the following problems, substitute 10 for a.

Problem	Calculation	Solution
1. $a+1$	Replace the a with 10. $10+1$	11
2. $17-a$	$17-10$	7
3. $9a$	This means multiply. 9×10	90
4. $\frac{30}{a}$	This means divide. $30 \div 10$	3
5. a^3	$10 \times 10 \times 10$	1,000
6. $5a+6$	$(5 \times 10)+6$	56

Note: Be sure to do all multiplying and dividing before adding and subtracting.

Example 2: In the following problems, let $x=2$, $y=4$, and $z=5$.

Problem	Calculation	Solution
1. $5xy+z$	$5 \times 2 \times 4+5$	45
2. xz^2+5	$2 \times 5^2+5=2 \times 25+5$	55
3. $\frac{yz}{x}$	$(4 \times 5) \div 2=20 \div 2$	10

In the following problems, $t=7$. Solve the problems. (DOK 2)

1. $t+3=$
2. $18-t=$
3. $\frac{21}{t}=$
4. $3t-5=$
5. $t^2+1=$
6. $2t-4=$
7. $9t \div 3=$
8. $\frac{t^2}{7}=$
9. $5t+6=$
10. $\frac{(t^2-7)}{6}=$
11. $4t+5t=$
12. $\frac{6t}{3}=$

In the following problems $a=4$, $b=2$, $c=5$, and $d=10$. Solve the problems. (DOK 2)

13. $4a+2c=$
14. $3bc-d=$
15. $\frac{ac}{d}=$
16. $d-2a=$
17. $a^2-b=$
18. $abd=$
19. $5c-a=$
20. $cd+bc=$
21. $\frac{6b}{a}=$
22. $9a+b=$
23. $5+3bc=$
24. $d^2+d+1=$

8.2 Substituting Numbers for Variables (DOK 1, 2)

These problems may look difficult at first glance, but they are very easy. Simply replace the variable with the number the variable is equal to, and solve the problems.

Example 1: In the following problems, substitute 3 for b.

Problem	Calculation	Solution
1. $b+2$	Simply replace the b with 3. $3+2$	5
2. $13-b$	$13-3$	10
3. $4b$	This means multiply. 4×3	12
4. $\frac{27}{b}$	This means divide. $27\div 3$	9
5. b^3	$3\times 3\times 3$	27
6. $4b+8$	$(4\times 3)+8$	20

Note: Be sure to do all multiplying and dividing before adding and subtracting.

Example 2: In the following problems, let $a=3$, $b=-1$, and $c=4$.

Problem	Calculation	Solution
1. $3ac-b$	$3\times 3\times 4-(-1)$	37
2. cb^2+2	$4\times(-1)^2+2=4\times 1+2$	6
3. $\frac{ac}{3}$	$(3\times 4)\div 3=12\div 3$	4

In the following problems let $r=9$. Solve the problems. (DOK 1)

1. $r-3=$
2. $11+r=$
3. $\frac{63}{r}=$
4. $r^2+1=$
5. $4r-6=$
6. $r^2-80=$
7. $\frac{r^2}{9}=$
8. $3r+4=$
9. $6r\div 2=$

In the following problems let $r=-4$, $s=10$, $t=2$, and $u=-1$. Solve the problems. (DOK 2)

10. $-\frac{1}{4}r+\frac{1}{8}t=$
11. $\frac{tu}{9}=$
12. $s-7=$
13. $2ut+r=$
14. $\frac{3t}{2}=$
15. $u^2+s+1=$
16. $3(4+t)=$
17. $s-9+t=$
18. $\frac{3}{2}r^2+4=$
19. $rtu=$
20. $3s+\frac{1}{5}sr=$
21. $rs+tu=$

8.3 Substituting Numbers in Formulas (DOK 2)

Many formulas are used in mathematics. There are formulas for finding the surface area of an object, the volume of an object, the rate of speed of a vehicle, etc. When describing the measures used in a formula follow these rules:

The variable x^2 is read x squared.	**Example:** Stacey's paper has an area of 68 in^2.
The variable x^3 is read x cubed.	**Example:** Robert's cylinder has a volume of 122 in^3.

In the problems below, a formula and the value of the variable in the formula will be given. Substitute the value of the variable in each formula and solve. The first one is done for you. (DOK 2)

1. The formula for finding the volume of a cube is $V = s^3$, where s is the length of the side of a cube. Terrance has a cube that measures 3 inches along each side. What is the volume of Terrance's cube?

   $V = 3^3 = 27\ in^3$

2. The formula for the area of a rectangular piece of paper is $A = lw$. The length, l, of Lizetta's piece of paper is 6 inches. The width, w, of her paper is 4 inches. What is the area of Lizetta's piece of paper in square inches?

3. The formula for the volume of a rectangular prism is $V = lwh$. Maurice has a box that measures $l = 14$ inches, $w = 6$ inches, and $h = 4$ inches. What is the volume of Maurice's box? Express your answer in cubic inches.

4. The formula for the surface area of a cube is $s^2 \times 6$. Nicole has a cube that measures 10 centimeters on each edge. What is the volume of Nicole's cube? Express your answer in square centimeters.

5. The formula for the area of a rectangular piece of cardboard is $A = lw$. The length, l, of Elijah's piece of cardboard is 25 cm. The width, w, of his cardboard is 10 cm. What is the area of Elijah's piece of cardboard in square cm?

6. The formula for the volume of a pyramid is $V = \frac{1}{3}Bh$. Grace has a pillow shaped like a pyramid. The base is 144 square inches. The height is 10 inches. What is the volume of Grace's pillow shaped like a pyramid in cubic inches?

7. The formula for the volume of a rectangular prism is $V = lwh$. A box of cereal has measurements of $l = 11$ inches, $w = 3$ inches, and $h = 13$ inches. What is the volume of this box of cereal in cubic inches?

8.3 Understanding Algebra Word Problems (DOK 1, 2)

The biggest challenge to solving word problems is figuring out whether to add, subtract, multiply, or divide. Below is a list of key words and their meanings. This list does not include every situation you might see, but it includes the most common examples.

Words Indicating Addition	Example	Add
and	3 **and** 9	$3 + 9$
increased	The original price of \$14 **increased** by \$2.	$14 + 2$
more	7 coins and 3 **more**	$7 + 3$
more than	Josh has 15 points. Will has 3 **more than** Josh.	$15 + 3$
plus	2 baseballs **plus** 1 baseballs	$2 + 1$
sum	the **sum** of 4 and 2	$4 + 2$
total	the **total** of 9, 5, and 11	$9 + 5 + 11$

Words Indicating Subtraction	Example	Subtract
decreased	\$19 **decreased** by \$7	$19 - 7$
difference	the **difference** between 24 and 10	$24 - 10$
less	12 days **less** 5	$12 - 5$
less than	Jose completed 11 laps **less than** Mike's 15.	$*15 - 11$
left	Ray sold 22 out of 40 tickets. How many did he have **left**?	$*40 - 22$
lower than	This month's rainfall is 3 inches **lower than** last month's rainfall of 9 inches.	$*9 - 3$
minus	8 **minus** 7	$8 - 7$

* In subtraction word problems, you cannot always subtract the numbers in the order that they appear in the problem. Sometimes the first number should be subtracted from the last. You must read each problem carefully.

Words Indicating Multiplication	Example	Multiply
triple	Her \$150 profit **tripled** in in a month.	150×3
half	**Half** of the \$800 collected went to charity.	$\frac{1}{2} \times 800$
product	the **product** of 5 and 11	5×11
times	Li scored 5 **times** as many points as Ted who only scored 3.	5×3
double	The bacteria **doubled** its original colony of 5,000 in just one day.	$2 \times 5,000$
twice	Ron has 8 CDs. Tom has **twice** as many.	2×8

Words Indicating Division	Example	Divide
divide into, by, or among	The group of 20 **divided into** 5 teams	$20 \div 5$ or $\frac{20}{5}$
quotient	the **quotient** of 36 and 4	$36 \div 4$ or $\frac{36}{4}$

Match the phrase with the correct algebraic expression below. The answers will be used more than once. (DOK 1)

A. $x + 4$ B. $4x$ C. $4 - x$ D. $x - 4$ E. $\frac{x}{4}$

1. 4 more than x
2. x divided into 4
3. 4 less than x
4. four times x
5. the quotient of x and 4
6. x increased by 4
7. 4 less x
8. the product of 4 and x
9. x decreased by 4
10. x times 4
11. 4 minus x
12. the total of 4 and x

Now practice writing parts of algebraic expressions from the following word problems. (DOK 2)

Example 3: the product of 3 and a number, x Answer: $3x$

13. the sum of 3 and y
14. x minus 2
15. the quotient of r divided by 7
16. 5 more than p
17. 2 less than y
18. triple n
19. the total of h and 14
20. 7 less r
21. double y
22. 2 increased by c
23. 8 less than z
24. half of r
25. 4 times t
26. z minus 5
27. 8 plus m
28. 3 divided by s
29. the product of 4 and n
30. z decreased by 10
31. four times as much as x
32. q less than 12

Write an algebraic expression for each word problem below. (DOK 1, 2)

9. five guests more than planned

10. the class with eight students missing

11. a number decreased by thirty-one

12. the difference of a number and eighteen

13. Eight dollars per hour

14. the product of eight and the third power of a number

15. nine dollars minus purchases

16. eighty percent of a number

17. the total number of cupcakes divided among four trays

18. half the number of cookies plus seven extra

19. bacteria culture, b, doubled

20. triple John's age, y

21. n feet lower than 10

22. 3 more than p

23. the product of 4 and m

24. a number, y, decreased by 20

25. 5 times as much as x

26. a number, n, plus 4

27. quantity, t, less 6

28. 18 divided by a number, x

If a word problem contains the word "sum" or "difference," put the numbers that "sum" or "difference" refer to in parentheses to be added or subtracted first. Do not separate them. Look at the examples below.

Examples:

	RIGHT	WRONG
sum of 2 and 4, times 5	$5(2+4)=30$	$2+4\times 5=22$
the sum of 4 and 6, divided by 2	$\frac{(4+6)}{2}=5$	$4+\frac{6}{2}=7$
4 times the difference between 10 and 5	$4(10-5)=20$	$4\times 10-5=35$
20 divided by the difference between 4 and 2	$\frac{20}{(4-2)}=10$	$20\div 4-2=3$
the sum of x and 4, multiplied by 2	$2(x+4)=2x+8$	$x+4\times 2=x+8$

Change the following phrases into algebraic expressions. (DOK 2)

1. 4 times the sum of x and 2
2. the difference between 8 and 4, divided by 2
3. 60 divided by the sum of 5 and 2
4. twice the sum of 15 and x
5. the difference between x and 7, divided by 3
6. 6 times the difference between x and 3
7. 10 multiplied by the sum of 4 and 5
8. the difference between x and 3, divided by 5
9. x divided by the sum of 7 and 2
10. x minus 3, times 7
11. 70 multiplied by the sum of x and 4
12. twice the difference between 4 and x
13. 8 times the sum of 2 and 9
14. 3 times the difference between 8 and 1
15. 14 divided by the sum of 3 and 11
16. four minus x, multiplied by 15

Look at the examples below for more phrases that may be used in algebra word problems.

Examples

one-half of the sum of x and 4	$\frac{1}{2}(x+4)$ or $\frac{x+4}{2}$
six more than four times a number, x	$6+4x$
100 decreased by the product of a number, x, and 5	$100-5x$
ten less than the product of 3 and x	$3x-10$

Change the following phrases into algebraic expressions. (DOK 2)

1. one-third of the sum of x and 2
2. three more than the product of a number, x, and 5
3. ten less than the sum of t and 8
4. the product of 2 and n, minus 8
5. 9 less than the sum of 4 and x
6. the difference of the numbers 16 and 12, times a number, n
7. one-eighth of t
8. the product of 4 and x, minus 9
9. x times the difference between 2 and x
10. five plus the quotient of x and 9
11. the sum of 8 and k, divided by 3
12. one less than the product of 7 and x
13. 4 increased by one-half of a number, n
14. 25 more than twice x
15. seven subtracted from four times m
16. 9 times x, subtracted from 13

8.4 Setting Up Algebra Word Problems (DOK 2)

To complete an algebra problem, an equal sign must be added. The words "**is**" or "**are**" as well as "**equal(s)**" signal that you should add an equal sign.

Example 3: Double Jake's age, x, minus 4 is 22.

$$2x - 4 = 22$$

Translate the following word problems into algebra problems. DO NOT find the solutions to the problems yet. (DOK 2)

1. Triple the original number, n, is 3,500.
2. The product of a number, y, and 3 is equal to 15.
3. Four times the difference of a number, x, and 10 is 35.
4. The total, t, divided into 7 groups is 20.
5. The number of parts in inventory, p, minus 80 parts sold today is 270.
6. One-half an amount, x, added to \$45 is \$342
7. One hundred seeds divided by 4 rows equals n number of seeds per row.
8. A number, y, less than 20 is 47.
9. His base pay of \$500 increased by his commission, x, is \$640.
10. Seventeen more than half a number, h, is 100.
11. This month's sales of \$2,900 are double January's sales, x.
12. The quotient of a number, w, and 8 is 24.
13. Six less a number, d, is 32.
14. Four times the sum of a number, y, and 7 is 84.
15. We started with x number of students. When 2 moved away, we had 28 left.
16. A number, b, divided by 29 is 3.

8.5 Changing Algebra Word Problems to Algebraic Equations (DOK 2)

Example 4: There are 3 people who have a total weight of 595 pounds. Sally weighs 20 pounds less than Jessie. Rafael weighs 15 pounds more than Jessie. How much does Jessie weigh?

Step 1: Notice everyone's weight is given in terms of Jessie. Sally weighs 20 pounds less than Jessie. Rafael weighs 15 pounds more than Jessie. First, we write everyone's weight in terms of Jessie, j.

$$\begin{aligned} \text{Jessie} &= j \\ \text{Sally} &= j - 20 \\ \text{Rafael} &= j + 15 \end{aligned}$$

Step 2: We know that all three together weigh 595 pounds. We write the sum of everyone's weight equal to 595.

$$j + j - 20 + j + 15 = 595$$

We will learn to solve these problems in chapter 9.

Change the following word problems to algebraic equations. (DOK 2)

1. Fluffy, Spot, and Shampy have a combined age in dog years of 82. Spot is 14 years younger than Fluffy. Shampy is 6 years older than Fluffy. What is Fluffy's age, f, in dog years?

2. Jerry Marcosi puts 8% of the amount he makes per week into a retirement account, r. He is paid \$12.00 per hour and works 40 hours per week for a certain number of weeks, w. Write an equation to help him find out how much he puts into his retirement account.

3. A furniture store advertises a 35% off liquidation sale on all items. What would the sale price (p) be on a \$2,742 dining room set?

4. Kyle Thornton buys an item which normally sells for a certain price, x. Today the item is selling for 25% off the regular price. A sales tax of 8% is added to the equation to find the final price, f.

5. Tamika Francois runs a floral shop. On Tuesday, Tamika sold a total of \$800 worth of flowers. The flowers cost her \$75, and she paid an employee to work 8 hours for a given wage, w. Write an equation to help Tamika find her profit, p, on Tuesday. (This profit will be adjusted as Tamika will still need to pay herself, rent, utilities, etc. from the sales for the day.)

6. Sharice is a waitress at a local restaurant. She makes an hourly wage of \$2.70, plus she receives tips. On Monday, she works 8 hours and receives tip money, t. Write an equation showing what Sharice makes on Monday, y.

7. Jenelle buys x shares of stock in a company at \$27.80 per share. She later sells the shares at \$41.29 per share. Write an equation to show how much money, m, Jenelle has made.